AMERICAN UNIVERSITY OF BEIRUT

MATH 233, Final Exam

June 9, 2001

Time = 1 Hour and 50 Minutes

1. If $P(A) = 1/3$ and $P(B^*) = 1/4$, can A and B be disjoint? Explain. (5 pts)

2. Let $f(x) = (1/\beta)|1 - |(x - \alpha)/\beta||$ If $\alpha - \beta < x < \alpha + \beta$ where $-\infty < \alpha < +\infty$ and $\beta > 0$.

   (a) Show that $f(x)$ is a probability density function. (5 pts)

   (b) Find the distribution function that corresponds to $f(x)$. (10 pts)

   (c) Find the mean and variance of X. (5 pts)

3. X is a binomial random variable with $n = 1$ and probability of success p. Further, the random variable Y has $E(Y|X = 0) = 1$ and $E(Y|X = 0) = 2$, what is $E(Y)$? (5 pts)

4. Let X and Y have the joint probability density function

$$f(x, y) = k \text{ if } 0 < x < 1 \text{ and } 0 < y < 1 \tag{1}$$

   (a) What is the value of k for which $f(x, y)$ is a probability density function? (5 pts)

   (b) What is the distribution of $Z = X + Y$? (10 pts)

5. Let X and Y have the joint probability density function

$$f(x, y) = e^{-x-y} \text{ if } 0 < X < \infty \text{ and } 0 < y < \infty \tag{2}$$

   What is the distribution of the random variable $Z = X/Y$? (10 pts)

6. Let X_1 and X_2 be a random sample of size 2 from a distribution with a probability density function $f(x) = x^{-2}$ if $x > 1$. Let $Y = min\{X_1, X_2\}$.

   (a) Does $E(X_i)$, $i = 1, 2$, exist? If so, find it. (5 pts)

   (b) Does $E(Y)$ exist? If so, find it. (10 pts)

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